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HEAT CONDUCTION AND HEAT EXCHANGE IN TECHNOLOGICAL PROCESSES

NONSTATIONARY HEAT AND MASS TRANSFER IN EVAPORATIVE COOLING OF FLOWING LIQUID FILMS

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A mathematical model of nonstationary evaporative cooling of a laminar liquid film flowing down a vertical surface in its blowing with a countercurrent steam-air flow has been developed. The problem of heat and mass transfer has been formulated in a conjugate statement. The calculated data on the time change in the temperature and concentration fields in the steam-air flow and the liquid film as well as in the density of the heat flux on the flowing-film surface have been given.

The operating efficiency of thermal and nuclear power stations is largely determined by the efficiency of cooling of a circulating water in chimney-type evaporative cooling towers, where the evaporation of the liquid from the surface of gravitational films flowing down the shields of a wetting device is of primary importance for reduction in the temperature of water arriving from the turbine condenser. Furthermore, if we take into account the capital outlays for development and building of such megastructures, intensification of the processes of cooling in chimney-type evaporative cooling towers finally ensures a considerable economy of financial funds and material resources.

The nonstationary regime of wetting [1–3] is the most promising for increasing the cooling power of chimney-type evaporative cooling towers. However, to implement such an intensification method in practice one must know the regularities of nonstationary processes of evaporation and heat transfer in gravitational flow of a film.

Theoretical and experimental investigation of hydrodynamics and heat exchange in gravitational film flow of a liquid is the focus of numerous domestic and foreign works (see, e.g., [4–14]). In such works, consideration has been mainly given to the stationary regimes of heat and mass transfer, where the regularities of heat and mass exchange are determined experimentally or from calculations based on empirical dependences for the coefficients of heat and mass exchange. A one-dimensional mathematical model of combined cooling of water in a cooling tower due to both evaporative film cooling and the cooling of water droplets in the wetting zone has been proposed in [15]. An expression for determination of the thermal efficiency of the cooling tower, which is in proportion to the ratio of the specific flow rate of air and water and depends on the dimensionless parameter $\overline{P} = h^2V_\infty/(D_{12}L)$, has been obtained.

Analogous results have been obtained in [16], where one-dimensional and two-dimensional models of stationary heat and mass transfer in the case of flow of a steam-air mixture past a film have been considered and the range of applicability of a one-dimensional model in a film-type heat exchanger has been determined. The boundary conditions (temperature and density of a steam near the wall) for solution of a two-dimensional problem are taken from the solution of the corresponding one-dimensional problem.

The present work seeks to numerically model nonstationary processes of evaporation and heat transfer from the surface of a gravitational liquid film as applied to the cooling of a circulating water on the shields of a wetting device in chimney-type evaporative cooling towers.

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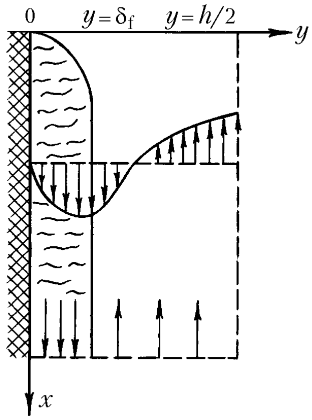

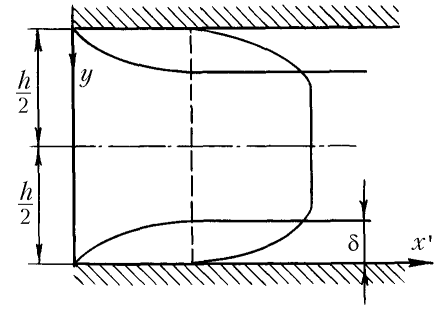

Fig. 1. Velocity profiles in the liquid and gas phases in the counterflow regime.

Fig. 2. Diagram of flow on the initial portion.

Statement of the Problem. We formulate a system of equations and boundary conditions for the nonstationary problem of heat and mass exchange in evaporative cooling of a flowing film by the counterflow of a steam-air mixture (Fig. 1). We will assume that the flow of the film is laminar and the distribution of the liquid velocity in it is known from the solution of an individual problem [17]:

$$V_f = \frac{\rho_f g \delta_f y}{\mu_f}\left(1 - \frac{y}{2\delta_f}\right). \tag{1}$$

The hydrodynamic problem in the gas phase has been solved with allowance for the formation of a velocity profile on the initial hydrodynamic portion. A similar problem for flow in a plane channel was considered in [18] and later in [19, 20]. A simple approximate solution of this problem can be obtained based on the notion of a boundary layer (this notion was used for the first time for a circular pipe in [21]).

We will assume that the velocity in the inlet cross section of the channel is constant and equal to V_∞. A boundary layer which develops in the same manner as that on the plate is formed on both channel walls due to friction. As a result, the channel flow is broken into three zones: the central zone, in which the liquid moves with a velocity identical over the cross section (flow core) and two lateral zones formed by the boundary layers (Fig. 2).

We assume that the thickness of the boundary layer along the pipe increases, just as on a plane plate. Assuming that μ is temperature-independent, to determine the boundary-layer thickness we obtain the expression

$$\delta = k\sqrt{\frac{\mu x'}{\rho V_\infty}},$$

where k is the arbitrary coefficient determined from the condition according to which, at the end of the initial hydrodynamic portion l_{in} equal to (see [22])

$$l_{in} = 0.04\,\frac{h^2 \rho V_\infty}{\mu},$$

we have the joining of the boundary layers, i.e., $\delta(x') = h/2$. As a result, we obtain

$$k = \frac{h}{2}\sqrt{\frac{\rho V_\infty}{\mu l_{in}}} = \sqrt{\frac{1}{0.16}} = 2.5\,.$$

The velocity in the flow core will be assumed to be constant over the cross section and to be equal to $3V_{av}/2$, whereas the velocity in the boundary layer will be considered to be distributed by the law [21]

$$V_x(x', y) = \begin{cases} \frac{3}{2} V_{av}, & \delta(x') < y < \frac{h}{2}; \\ \frac{3}{2} V_{av}(x') \left[1 - \frac{(y - \delta(x'))^2}{\delta^2(x')} \right], & 0 \le y \le \delta(x'). \end{cases} \tag{2}$$

A relationship between $V_{av}(x')$ and $\delta(x')$ is established from the condition of constancy of the volumetric flow rate of the liquid

$$\int_0^{\delta} V_x(x', y)\, dy + \frac{3}{2} V_{av}(x') \left(\frac{h}{2} - \delta \right) = \frac{h}{2} V_\infty ,$$

whose integration yields

$$V_{av} = \frac{hV_\infty}{3\frac{h}{2} - \delta(x')} .$$

The pressure distribution on the initial portion of the channel is found from the condition of equilibrium of a liquid layer of thickness dx':

$$\frac{\partial P}{\partial x'} = \frac{2\mu}{h} \left. \frac{\partial V_x}{\partial y} \right|_{y=0} .$$

After substituting $\frac{\partial V_x}{\partial y}$ into this expression from (2), to determine the pressure we obtain the formula

$$P = 0.96\rho V_\infty^2 \ln \left[1 - \frac{0.8333}{a} \sqrt{\frac{\mu x'}{\rho V_\infty}} \right] + P_0 .$$

Thus, the distribution of the vertical velocity component V_x and the pressure gradient dP/dx' for the flow of the steam-air mixture are known.

To describe the process of heat and mass exchange in the film–steam-gas flow system we use the following equations. If the longitudinal transfer of heat in the film is disregarded, the heat-transfer equation for it will take the form

$$\frac{\partial T_f}{\partial t} + V_{xf} \frac{\partial T_f}{\partial x} = \frac{\partial}{\partial y} \left(a_f \frac{\partial T_f}{\partial y} \right). \tag{3}$$

For the gas phase, which represents a binary gas mixture of air and steam, with account for the expression for the velocity in the gas flow (2) we write

$$\rho \left(\frac{\partial C}{\partial t} + V_x \frac{\partial C}{\partial x} \right) = \frac{\partial}{\partial y} \left(\rho D_{12} \frac{\partial C}{\partial y} \right), \tag{4}$$

$$\rho c_P \left(\frac{\partial T}{\partial t} + V_x \frac{\partial T}{\partial x} \right) = \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right). \tag{5}$$

Using the Dalton law, we can represent the equation of state and the thermodynamic parameters of the gas mixture as follows:

$$P = P^{(1)} + P^{(2)} = \rho R\,(T + 273) = \rho\,(T + 273) \sum_{i=1}^{2} C^{(i)} R^{(i)}\,, \quad \rho = \rho^{(1)} + \rho^{(2)}\,,$$

$$c_P = C c_P^{(1)} + (1 - C)\, c_P^{(2)}\,. \tag{6}$$

To obtain a unique solution of the system of equations (3)–(6) we must formulate the corresponding number of boundary and initial conditions, i.e., must have a closed conjugate statement of the problem.

The general principle of derivation of boundary conditions at the interface (as on strong-discontinuity surfaces) has been presented in [23] and has been used in [24–26]. Following these works and as applied to the problem in question, we write the conditions at the phase boundary $y = \delta_f(t)$ which have been obtained from the conservation laws:

$$y = \delta_f\,(t)\,, \quad -\rho_f \frac{d\delta_f}{dt} = \rho V_y\,, \tag{7}$$

$$V_y\,(1 - C) = -D_{12} \frac{\partial C}{\partial y}\,, \tag{8}$$

$$-\lambda_f \frac{\partial T_f}{\partial y} + \lambda \frac{\partial T}{\partial y} = -\rho_f \frac{d\delta_f}{dt}\, q = -q \frac{\rho D_{12}}{1 - C} \frac{\partial C}{\partial y}\,. \tag{9}$$

It is noteworthy that, in expression (9), we use the conditions of conservation of mass of the entire mixture (7) and the steam (8) to eliminate the velocity of movement of the film in the transverse direction $d\delta_f/dt$ because of the evaporation of the liquid from direct consideration in the problem. Therefore, we approximately assume that δ_f = const. Also, we do not use the equality of tangential stresses, since flow in the film is assumed to be one-dimensional. In addition to the conditions following from the conservation laws, at the evaporation boundary, we must specify supplementary ones reflecting the adhesion conditions, the equality of temperatures, and the kinetics of evaporation:

$$V_f = V_x\,, \tag{10}$$

$$T_f = T = T_e\,(C_e)\,. \tag{11}$$

It follows from (10) that the distribution (2) must be referred to a coordinate system moving together with the external boundary of the liquid film. According to (11), we assume that evaporation from the film surface is equilibrium, i.e., a supplementary relationship is the steam-elasticity curve (dependence of the density of a saturated steam on the temperature of the evaporation surface) approximated, from the data of [27], by the dependence

$$\rho\,(T) = 8.155 \cdot 10^{-7} T^3 - 3.604 \cdot 10^{-5} T^2 + 1.355 \cdot 10^{-3} T + 0.0007\,. \tag{12}$$

On the axis of symmetry of the channel, when $y = h/2$ we have

$$\frac{\partial C}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial V_x}{\partial y} = 0\,, \tag{13}$$

for all x, and when $y = 0$ we have

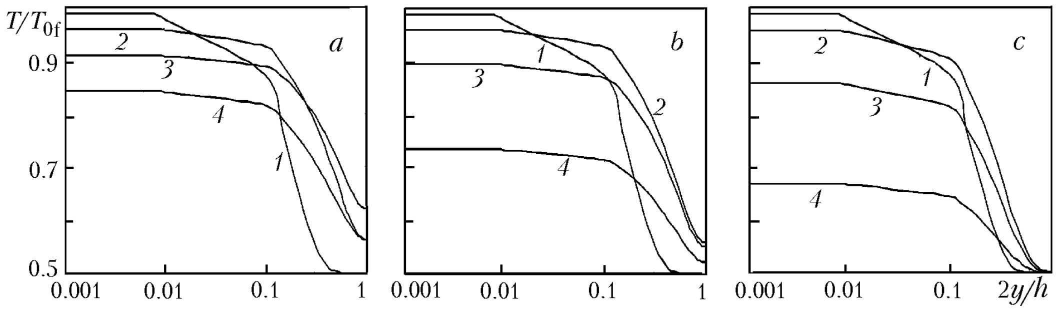

Fig. 3. Temperature distribution in the cross sections of the channel in a liquid film and in a steam-air medium at different instants of time for Re = 30.1: a) $x = L/6$; b) $L/2$; c) $5L/6$ [1) $t/\tau = 0.04$; 2) 0.4; 3) 2; 4) 8].

$$\frac{\partial T_f}{\partial y} = 0 . \tag{14}$$

Thus, for the system of differential equations (3)–(6) we have formulated boundary conditions (9)–(11), (13), and (14) for the variable y.

At the boundary $x = 0$, we specify:

$$T_f = T_{0f} , \quad \rho_f V_f = Q_{0f}(t) . \tag{15}$$

At the channel inlet ($x = L$), boundary conditions have the following form:

$$C = C_\infty , \quad V_x = V_\infty , \quad \rho = \rho_\infty , \quad T = T_\infty . \tag{16}$$

Initial conditions ($t = 0$) are as follows:

$$T_f = T_{0f} , \quad C = C_\infty , \quad T = T_\infty , \quad V_x = V_\infty .$$

The system of equations (3)–(6) obtained after substituting (1) and (2) (with $L - x$ in place of x') has been solved numerically by the finite-difference method with account for boundary conditions (8)–(16). In solving, we used a nonstationary implicit finite-difference scheme of variable directions. The convective terms were approximated by one-sided upstream differences, whereas symmetric differences were used for approximation of the diffusion terms. The system of difference equations was solved by the marching method.

The algorithm of determination of the solution at a new time level involved the following steps. First we found the gas-phase temperature from Eq. (5). The air temperature at the boundary $y = \delta(t)$ was taken to be equal to the film temperature. Next we determined the heat flux from the film of the gas phase with allowance for the heat of evaporation of the liquid and found the temperature distribution in the film. Finally, we calculated the concentration of the steam in the gas phase from Eq. (4). We used a difference grid uniform in space in discretizing the computational domain. The time step was selected from conditions ensuring stability of the computations.

Calculation Results. Numerical calculations were performed for the most characteristic parameters corresponding to the operation of an industrial chimney-type evaporative cooling tower of wetting area 3200 m^2:

Temperature of the arriving water	40^oC
Temperature of the arriving air	20^oC
Relative humidity of the arriving air	60%
Air velocity at the channel inlet	1.0 m/sec
Channel length (height of the shields of a wetting device)	2.5 m
Channel half-width	0.0125 m

For the laminar regime of flow of the film, the Reynolds number was selected to be 30.1, which corresponds to a film thickness of $\delta_f = 10^{-4}$ m.

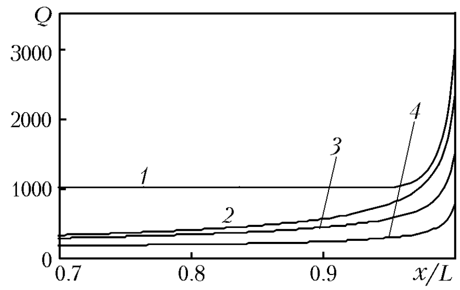

Fig. 4. Density of the total heat flux at the phase boundary at different instants of time for Re = 30.1: 1) t/τ = 0.04; 2) 0.4; 3) 2; 4) 8. Q, W/m^2.

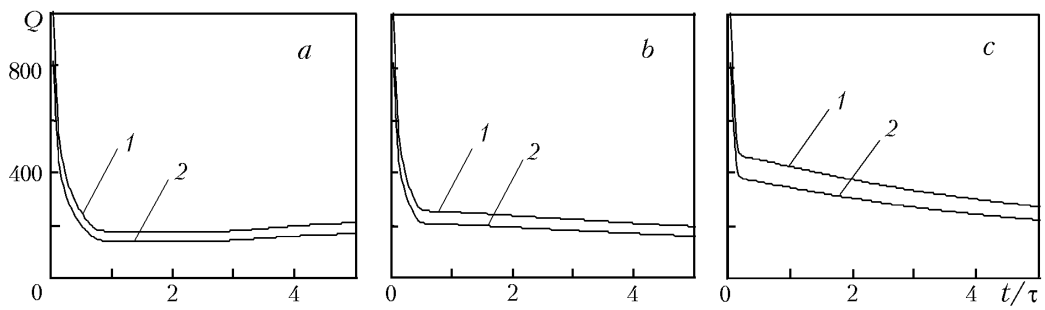

Fig. 5. Heat-flux densities vs. time for different portions along the channel length for Re = 30.1: a) $x = L/6$; b) $L/2$; c) $5L/6$ [1) total heat-flux density; 2) density of the heat flux due to evaporation].

The calculated values of the distributions of the temperature fields in the characteristic cross section of the channel in the liquid film and in the steam-air medium are presented in Fig. 3 in dimensionless form. The average time of traversal of the channel by the steam-gas flow $\tau = L/V_\infty$ is selected as the characteristic time.

A characteristic feature of the temperature distribution in the liquid film is the fact that the temperature is virtually constant over the film width in each cross section, except for the thin layer of the y boundary ($y = \delta_f$). In the cross section $x = 0$, as might be expected, the film temperature remains constant and equal to the arriving-liquid temperature. The film temperature is reduced with time with distance from the inlet portion. The intensity of cooling of the film increases with decrease in the Reynolds number.

The temperature of the steam-air medium monotonically changes with time in each cross section of the channel (except for $x = L$). The values of the temperature decrease with distance from the phase boundary to the axis of symmetry of the channel. The temperature of the steam-air medium grows with movement along the channel.

The plots of the change in the density of the total heat flux from the film surface at the phase boundary

$$Q = -\lambda_f \frac{\partial T_f}{\partial y}\bigg|_{y=\delta_f} = -\lambda \frac{\partial T}{\partial y}\bigg|_{y=\delta_f} - q \frac{\rho D_{12}}{1 - C} \frac{\partial C}{\partial y} ,$$

which correspond to the temperature fields given above, are presented in Fig. 4. As is seen in the figure, the value of the heat flux is maximum on the inlet portion on the source side of the arriving air ($x \approx L$). The change in the heat flux with time is also maximum here. Once the steady-state regime of heat exchange from the film surface has been reached with time, the heat flux decreases three to four times. What this means is that heat exchange from the film surface in the nonstationary regime exceeds heat removal in stationary flow of the film virtually along the entire length of the channel. Also, it is noteworthy that the zone of maximum change in the heat flux in the region of the inlet portion is about 1/10 of the channel length.

Figure 5 plots the changes in the total heat flux and the heat flux due to evaporation from the film surface for the inlet portion, at the center, and as the film leaves the channel. It is seen that cooling is mainly due to the evaporation from the film surface. On the inlet portion and in the central zone of flow, the fraction of the heat flux

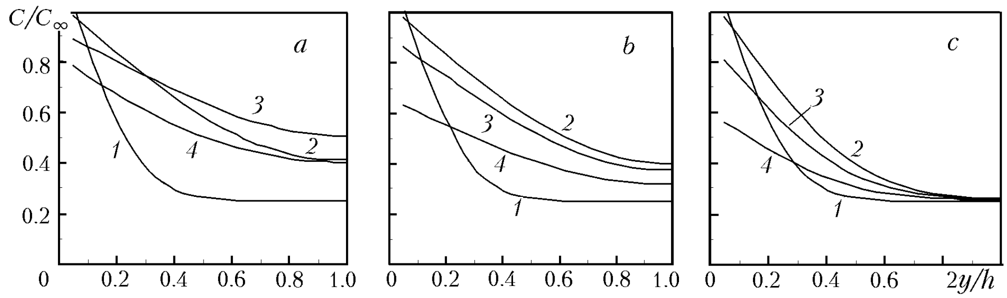

Fig. 6. Concentration distribution in the cross sections of the channel in a steam-air medium at different instants of time for Re = 30.1: a) $x = L/6$; b) $L/2$; c) $5L/6$. Notation 1–4 is the same as in Fig. 3.

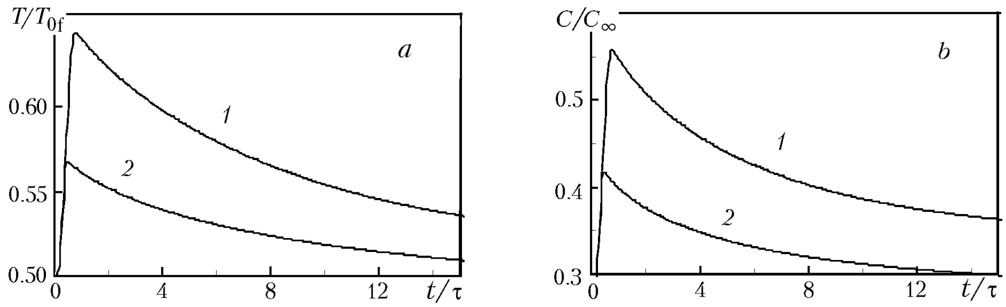

Fig. 7. Temperature (a) and concentration (b) on the channel axis ($y = h/2$) vs. time: 1) $x = L/2$; 2) $L/6$.

due to evaporation amounts to 85 to 90% of the total heat removal. The time of reaching the steady-state regime of heat exchange for these values of the Reynolds number is several tens of seconds for all the cross sections and is the characteristic time of spilling of water in a pulsating regime of wetting.

The concentration distribution of the steam in the characteristic cross sections of the channel in the steam-air medium at different instants of time is shown in Fig. 6. As might be expected, the concentration remains constant at the channel inlet in the direction of motion of the air ($x = L$) and equal to the concentration of moisture in the arriving air. The concentration of the steam increases with distance from the inlet portion, attaining the maximum values, as the steam-air medium leaves the channel. The value of the concentration, just as that of the temperature, decreases with distance from the phase boundary to the axis of symmetry of the channel.

Of interest is the nonmonotonic change in the temperature and concentration of the steam on the channel axis ($y = h/2$) with time (Fig. 7). Such behavior of these parameters is caused by the change in the temperature of the evaporating film in the process of heat and mass exchange. The processes of evaporation and heat transfer from the surface film are maximum at the initial instant of time. As the flowing film cools down and the thermal and diffusion boundary layers are formed, the intensity of evaporation and heat transfer decrease, which finally leads to a reduction in the temperature and concentration of the steam in the gas phase.

CONCLUSIONS

1. The conjugate problem of nonstationary heat and mass transfer in evaporative cooling of a flowing water film has been formulated and solved.

2. It has been shown that heat exchange from the film surface in a nonstationary regime exceeds heat removal in a stationary regime several times. The time of reaching the steady-state regime of heat exchange is five to ten characteristic times of traversal of the channel by the steam-gas flow.

3. The results of numerical modeling confirm the efficiency of application of nonstationary heat exchange to evaporative cooling of flowing water films.

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NOTATION

a, thermal diffusivity, m^2/sec; C, mass concentration of the steam; c_p, specific heat at constant pressure, J/(kg·^{o}C); D_{12}, coefficient of mutual diffusion, m^2/sec; g, free-fall acceleration, m/sec^2; h, channel width, m; L, channel length, m; l_{in}, length of the initial portion, m; P, pressure, Pa; Q, heat-flux density, W/m^2; q, specific heat of evaporation, J/kg; R, gas constant, J/(kg·^{o}C); $\mathrm{Re} = \frac{V_f \rho_f \delta_f}{\mu_f}$, Reynolds number; T, temperature, ^{o}C; t, time, sec; V, velocity, m/sec; x and y, longitudinal and transverse coordinates; δ_f, film thickness, m; λ, thermal conductivity, W/(m·^{o}C); μ, dynamic viscosity, Pa·sec; ρ, density of the steam-air mixture, kg/m^3; τ, characteristic time, sec. Subscripts: f, liquid film; ∞, entry of the steam-air flow into the channel; in, initial; av, average; e, on the saturation line.

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